UNIT 5 - COLLISIONS

1. Collision Types:

↳ Velcro (cars stick together), V_f is always slower than V_0

↳ Spring Collisions - if masses of the carts are the same, the carts swap V

2. Define Variables:

↳ Impulse = J → a force acting over a time: $J = (\sum F_{ext})(\Delta t)$ in Nsec ($kg\,m/s^2$) — they are the same

↳ Momentum = P → a measure of how difficult it is to stop something in motion: $P = mV$ in kgm/s

3. Make equations that link variables:

↳ Impulse Equation

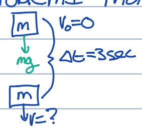
* $\Delta t(\sum F_{ext} = ma) = (\sum F_{ext})\Delta t = ma\Delta t \rightarrow J = ma\Delta t \rightarrow J = m(\frac{\Delta V}{\Delta t})(\Delta t) \rightarrow J = m\Delta V \rightarrow J = m(V - V_0) \rightarrow J = P - P_0 \rightarrow \boxed{J = \Delta P}$
* Reason - $J \rightarrow F \rightarrow a \rightarrow \Delta V \rightarrow \Delta P$

4. Problems with the impulse equation:

↳ $J = \Delta P \rightarrow (\sum F_{ext})(\Delta t) = mV - mV_0$

↳ must have $m, v, \Delta t, F$ in order to use Impulse Equation

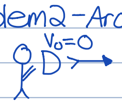
↳ Problem 1 - mass falling



$(3)(\cancel{m}g) = \cancel{m}V - \cancel{m(0)}$

$V = (3)(9.8) = \boxed{29.4\,m/s}$

↳ Problem 2 - Archery Problem

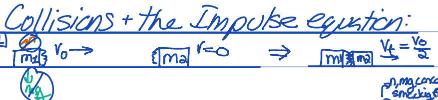


$(\sum F_{ext})(\Delta t) = mV - mV_0$

$\frac{(0.5)F = (0.018)(10)}{0.5} \Rightarrow F = \frac{(0.018)(10)}{0.5} = \boxed{0.36N}$

5. Collisions + the Impulse equation:

Case 1



N, mg cancel, sticking force internal: $P_f - P_i = 0$, so $P_f = P_i$

↳ Impulse equation: $J = \Delta P \rightarrow \cancel{(\sum F_{ext})(\Delta t)} = mV - mV_0$

↳ Famous Physics Theory - Conservation of P (momentum): $\boxed{\sum F_{ext} = 0 \rightarrow P_i = P_f}$

$(\cancel{m}V_0 + 0) = (2\cancel{m})V_f$

$\boxed{V_f = \frac{V_0}{2}\,m/s}$

Case 2

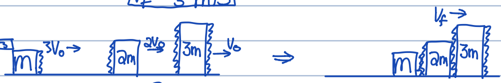
Q1: Is P conserved in this collision?

$\sum F_{ext} = 0 \rightarrow P_i = P_f$

$mV_0 + 0 = (3m)V_f$

$\boxed{V_f = \frac{V_0}{3}\,m/s}$

Case 3



Q1: What is the V_f?

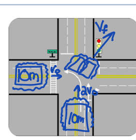
$\sum F_{ext} = 0 \rightarrow P_i = P_f$

$\cancel{m}3V_0 + (2\cancel{m})2V_0 + (3\cancel{m})V_0 = (6\cancel{m})V_f$

$10V_0 = 6V_f \Rightarrow \boxed{V_f = \frac{5V_0}{3}\,m/s}$

↳ Velcro Collisions in 2d

* Case 1



Q1: What is V_f?

$\sum F_{ext} = 0 \rightarrow P_i = P_f$

$P_{1i} = (10m)V_0$ \} tip to tail vector

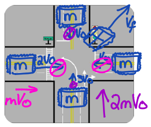
$P_{2i} = (10m)2V_0$

$P_{2i} = 20mV_0$, $P_i = 10mV_0$; $P_i = \sqrt{(10mV_0)^2 + (20mV_0)^2} = \sqrt{500m^2V_0^2} \rightarrow P_i = 22.36mV_0$

$P_i = 22.36mV_0 = (20m)V_f = P_f$

$V_f = \frac{22.36V_0}{20} \rightarrow \boxed{V_f = 1.118V_0 m/s}$

* Case 2 - 4 car, 2d pileup



Q: What is V_f?

$\sum F_{ext} = 0 \rightarrow P_i = P_f$

$P_{1i} = 2mV_0$

$P_{2i} = 3mV_0$

$P_{3i} = mV_0$

$P_{4i} = mV_0$

mV_0, $2mV_0$; $P_i = \sqrt{(mV_0)^2 + (2mV_0)^2} = \sqrt{5m^2V_0^2} = 2.24mV_0 \rightarrow P_i = 2.24mV_0$

$P_i = 2.24mV_0 = (4m)V_f = P_f \Rightarrow V_f = \frac{2.24V_0}{4} \Rightarrow \boxed{V_f = 0.56V_0 m/s}$

↳ 1D Explotions

Case 1: $V=0$ [m|m] $\Rightarrow$ $\overset{V_{2f}}{\leftarrow}$[m] [m]$\overset{V_{1f}}{\rightarrow}$

hypothesis: $V_1f = -V_2f$

$\sum F_{ext} = 0 \rightarrow P_i = P_f \rightarrow 0 = mV_1f + mV_2f \rightarrow \boxed{V_{2f} = -V_{1f}}$

Case 2: $V=0$ [m|2m] $\Rightarrow$ $\overset{V_{2f}}{\leftarrow}$[m] [2m]$\overset{V_{1f}}{\rightarrow}$

hypothesis: $2V_1f = -V_2f$

$\sum F_{ext} = 0 \rightarrow P_i = P_f \rightarrow 0 = 2mV_1f + mV_2f \rightarrow \boxed{2V_1f = -V_2f}$

Case 3: $V=0$ [m|3m] $\Rightarrow$ $\overset{V_{2f}}{\leftarrow}$[m] [3m]$\overset{V_{1f}}{\rightarrow}$

hypothesis: $3V_1f = -V_2f$

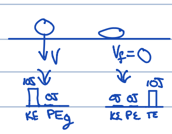
$\sum F_{ext} = 0 \rightarrow P_i = P_f \rightarrow 0 = 3mV_1f + mV_2f \rightarrow \boxed{3V_1f = -V_2}$

Case 4: V_0 [25kg] [5kg] $5V_0$

ratio of the masses gives the ratio of the final velocities: 1:5

6. Collisions + Energy:

I



\} KE was lost, became TE

↳ Inelastic Collision - a collision in which some KE is lost to TE. KE is not conserved

↳ Elastic Collision (springy collision) KE is conserved - ball bounces back

* bonds between atoms and molecules behave like springs in black ball that is springy

↳ Velcro Theorem - all velcro collisions are inelastic collisions (לֹא תָּמִיד)

↳ Velcro Collisions - find V_f (conservation of P)

↳ Inelastic Collisions

[m_1]$\overset{V_{1i}}{\rightarrow}$ $\overset{V_{2i}}{\leftarrow}$[$m_2$]

$\overset{V_{1f}}{\leftarrow}$[$m_1$] [$m_2$]$\overset{V_{2f}}{\rightarrow}$ \] classic

(spring) cons KE: $\frac{1}{2}m_1V_{1i}^2 + \frac{1}{2}m_2V_{2i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2$

($\sum F_{ext} = 0$) cons P: $m_1V_{1i} + m_2V_{2i} = m_1V_{1f} + m_2V_{2f}$

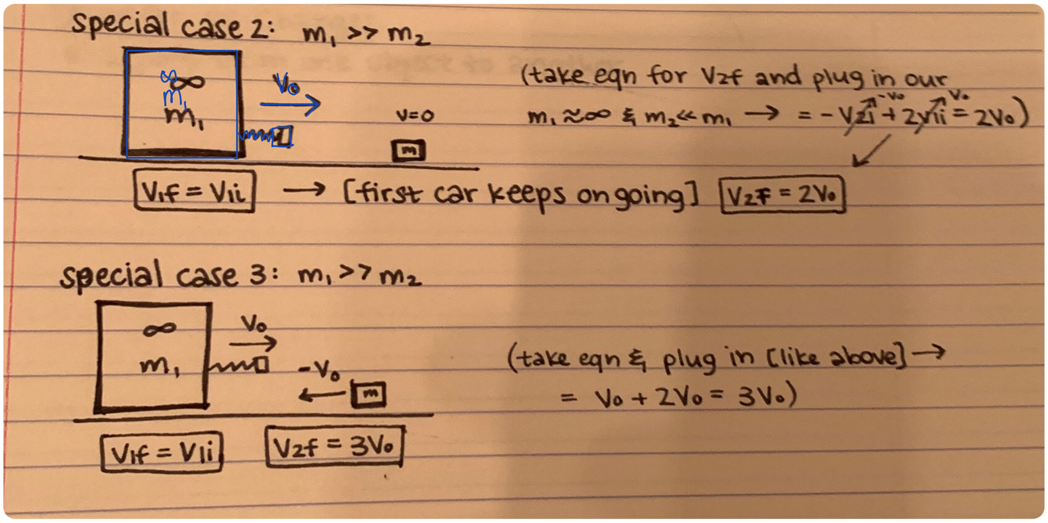
1d Elastic Collisions

$$\boxed{\begin{aligned} V_1f &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)V_{2i} \\ V_2f &= \left(\frac{m_2 - m_1}{m_2 + m_1}\right)V_{2i} + \left(\frac{2m_1}{m_2 + m_1}\right)V_{1i} \end{aligned}}$$

* When m's are equal, the cars swap velocities

Special case 1 - masses are equal

$$\left.\begin{aligned} V_1f &= \cancel{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)}^{0}V_{1i} + \cancel{\left(\frac{2m_2}{m_1 + m_2}\right)}^{1}V_{2i} \\ V_2f &= \cancel{\left(\frac{m_2 - m_1}{m_2 + m_1}\right)}V_{2i} + \cancel{\left(\frac{2m_1}{m_2 + m_1}\right)}V_{1i} \end{aligned}\right\} \boxed{\begin{aligned} V_1f &= V_{2i} \\ V_{2f} &= V_{1i} \end{aligned}}$$



Special case 2: $m_1 >> m_2$

V_0 $\quad v=0$

(take eqn for V_{2f} and plug in our $m_1 \approx \infty$ & $m_2 \ll m_1$ $\rightarrow$ $= -V_{2i} + 2V_{1i} = 2V_0$)

$\boxed{V_{1f} = V_{1i}}$ $\rightarrow$ [first car keeps on going] $\boxed{V_{2f} = 2V_0}$

Special case 3: $m_1 >> m_2$

V_0 $\quad -V_0$

(take eqn & plug in [like above] $\rightarrow$ $= V_0 + 2V_0 = 3V_0$)

$\boxed{V_{1f} = V_{1i}}$ $\boxed{V_{2f} = 3V_0}$

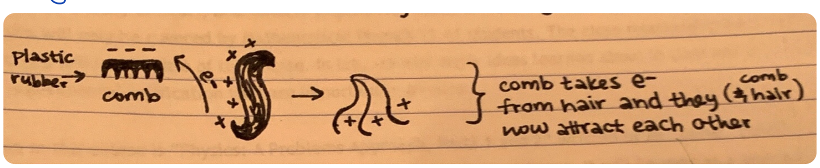
ELECTRICITY & MAGNETISM

1. Intro:

↳ Isaac Newton (1642-1727) - by the end of the 1600's, all of the Newton Laws were established

↳ electrons ⊖, protons ⊕ → not visible

↳ Ben Franklyn -

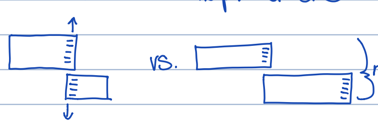


plastic rubber → comb

} comb takes e- from hair and they (comb & hair) now attract each other

↳ Transfer of charges

* movement of e's (not p) from one substance to another



vs. } nothing moves, charge stays

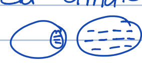
* Electrical insulator - a material that keeps charge in 1 place (usually non-polar organic molecules: plastic)

share electrons; polar - w/ high electronegativity

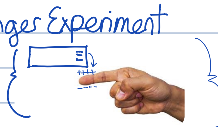
e.g. insulator

charge stays in one place

* Electrical conductor - a material that allows charge to spread on its surface (metals)



* Finger Experiment

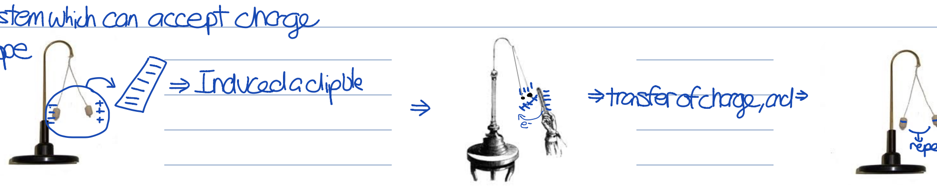


$F^{+/-}$ $F^{+/-}$

Induced a dipole - bring a charged object close to a neutral object → has a + end / - end, neutral object separates; dipoles are attracted to charged objects

* Electroscope - a system which can accept charge

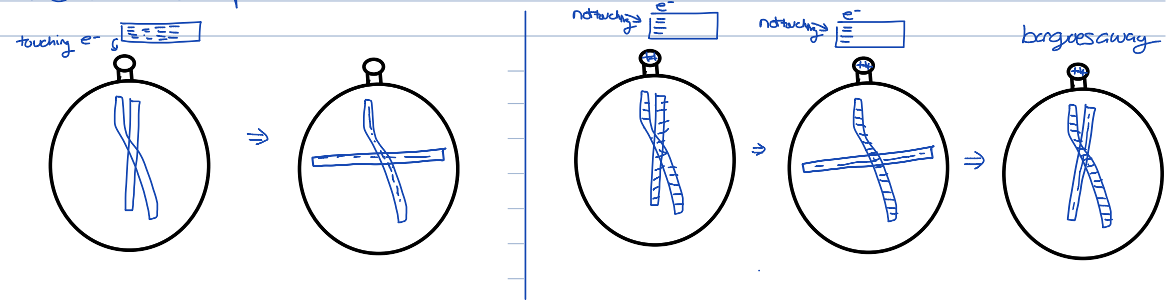
Ex: Piff electroscope

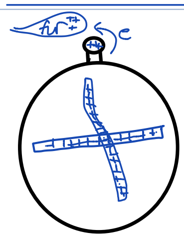


⇒ Induced a dipole ⇒ ⇒ transfer of charge, and ⇒

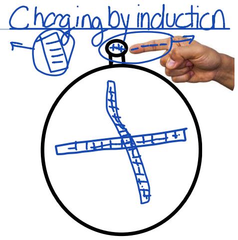
repel

Ex Metal Electroscope





Charging by induction



→ inducing a dipole

2. Charge:

↳ Introduction

(M) $\xrightarrow{F_g}$ $\xleftarrow{F_g}$ (m)

* N3 2Fg's are the same
* $F_g = \frac{GMm}{r^2}$, $G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$
* Fg is weak

↳ Electric

(+) $\xrightarrow{F_{el}}$ $\xleftarrow{F_{el}}$ (−)

* $\boxed{F_{el} = \frac{kq\overset{\text{charge}}{Q}}{r^2}}$ ← Coulomb's law (1785)

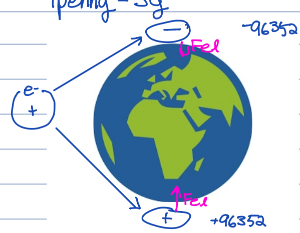
$\boxed{q,\text{proton} = 1.6\times10^{-19}\,C}$ $\boxed{m_p = 1.67\times10^{-27}\,kg}$

$\boxed{q,\text{electron} = -1.6\times10^{-19}\,C}$ $\boxed{m_e = 9.11\times10^{-31}\,kg}$

$1C = 10^{19}$ protons → very large amount of charge

* Coulombs (C) in metric system
  - milli Coulomb (mC) $= 10^{-3}C$
  - micro Coulomb (μC) $= 10^{-6}C$
  - nano Coulomb (nC) $= 10^{-9}C$
  - Pico Coulomb (PC) $= 10^{-12}C$
  - femto Coulomb (fC) $= 10^{-15}C$
* Hydrogen Problem

1 penny = 3g



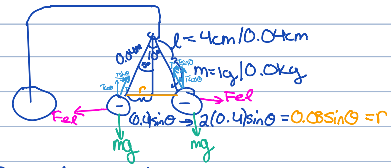
$Q_{1\,gram\,H} = (6.022\times10^{23}\,\text{Protons})\left(\frac{1.6\times10^{-19}C}{\text{Protons}}\right) = 96{,}352C$

$\rightarrow F_{el} = \frac{kqQ}{r^2} \rightarrow \frac{(9E9)(96352)(+96352)}{(\underbrace{2(6.38E6)}_{2r\,(\text{diameter})})^2} = \boxed{513171N}$ ← we want it positive

* 2 electrons problem

(get nitzy's pic)

* Pith ball Electroscope Problem



Step 1: FBD

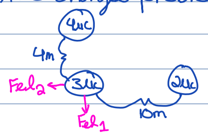
Step 2: N2 + solve

$\left.\begin{aligned}\frac{kq^2}{r^2} &= F_{el} = T\sin\theta \\ \div\; mg &= T\cos\theta\end{aligned}\right\}\ \frac{kq^2}{r^2 mg} = \tan\theta \rightarrow q = \sqrt{\frac{(\tan\theta) r^2 mg}{k}} \rightarrow q = \sqrt{\frac{(\tan 5)(0.08\sin 5)^2(0.001)(9.8)}{9E9}} \Rightarrow \boxed{q = 2.15E-9 \text{ Coulombs}}$

Q: Find how many electrons on the pith ball?

$\frac{\boxed{q = 2.15E-9C}}{1.6E-19C/e^-} \Rightarrow \boxed{1\times10^{10} e^-}$

* 3 charges problem - what is total force on the 3uC charge? (no gravity)

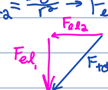


Step 1: FBD

Step 2/3: N2, solve

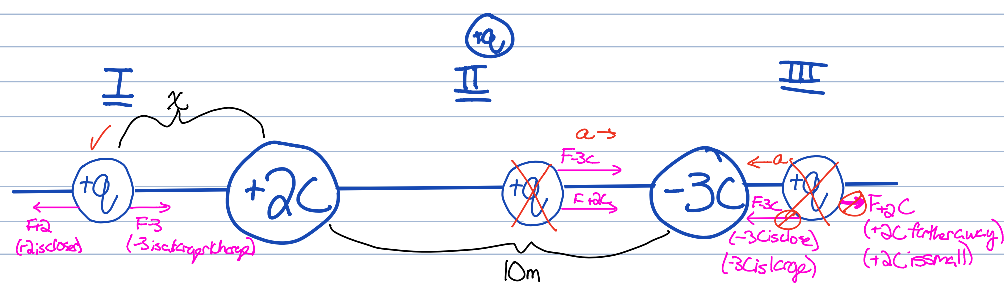
$F_{el_1} = \frac{kqQ}{r^2} \rightarrow F_{el_1} = \frac{(9E9)(4E-6)(3E-3)}{4^2} \rightarrow \boxed{F_{el_1} = 6.75N}$

$F_{el_2} = \frac{kqQ}{r^2} \rightarrow F_{el} = \frac{(9E9)(2E-6)(3E-3)}{10^2} \rightarrow \boxed{F_{el_2} = 0.54N}$



$F_{total} = \sqrt{F_{el_1}^2 + F_{el}^2} \rightarrow \boxed{F_{tot} = 6.77N}$

* Tug of War Problem - where should I put q charge so that it remains at rest?



Step 1: FBD

Step 2/3: N2 and Solve

$F_{+2} = F_{-3}$

$\left(\frac{\cancel{kq}Q}{r^2}\right)_{+2} = \left(\frac{\cancel{kq}Q}{r^2}\right)_{-3}$

$\frac{+2}{x^2} = \frac{3}{(10+x)^2}$

$\sqrt{2(10+x)^2} = \sqrt{3x^2}$

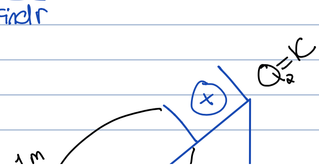
$\sqrt{2}(10+x) = x\sqrt{3}$

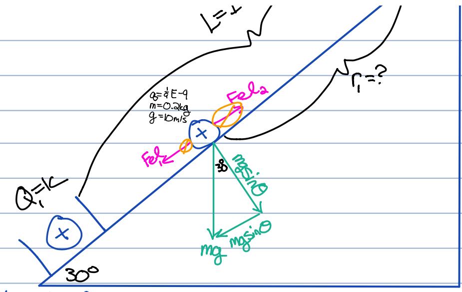
$10\sqrt{2} + x\sqrt{2} = x\sqrt{3}$

$10\sqrt{2} = x(\sqrt{3} - \sqrt{2})$

$x = \frac{10\sqrt{2}}{\sqrt{3}-\sqrt{2}} \longrightarrow \boxed{x = 44.49m}$

* Charge on a wedge - Find r





- Step 1: FBD
- Step 2/3: N2 + Solve

$$Fel_1 + mg\sin\theta = Fel_2$$

$$\frac{kqQ}{r_1^2} + mg\sin\theta = \frac{kqQ}{(1-r_1)^2}$$

$$r_1^2(1-r_1)^2\left(\frac{\cancel{(9E9)(4E-9)(1)}}{r_1^2}+\cancel{(0.2)(10)(\tfrac{1}{2})}=\frac{\cancel{(9E9)(4E-9)(1)}}{(1-r_1)^2}\right)$$

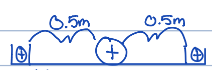
$$(1-r_1)^2 + r_1^2(1-r_1)^2 = r_1^2$$

$$(1-2r_1+r_1^2)+(\cancel{r_1^2}-2r_1^3+r_1^4)=\cancel{r_1^2}$$

$$r_1^4-2r_1^3+r_1^2-2r_1+1=0$$

$\boxed{r_1=0.531\,m}$ (must be between 0 and 1)

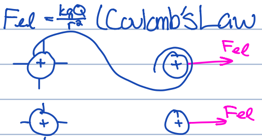
- Test answer



⇒ over 0.5 with mg → 0.531 > 0.5 ✓

↳ Electrical Field

$Fel=\frac{kqQ}{r^2}$ (Coulomb's Law - 1785



* Coulomb's Tentacle Theory = charge → charge as if it had a grip on it REJECTED

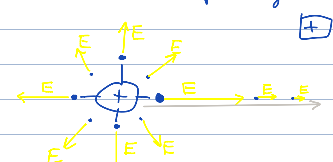
* Electric Field theory = a middle man intermediary that conveys the force information from charge to charge ✓

charge → E Field → charge

- Electric Field - a vector field a vector defined for every point in space
- Direction of E - direction a positive charge would move if it were placed in space
- Magnitude of E -

$Fel=\frac{kqQ}{r^2} \longrightarrow E_{point\,charge}=\frac{kQ}{r^2}$ where $E=\frac{Fel}{q}$ in $\frac{Newton}{Coulomb}$

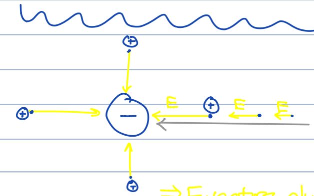
$\boxed{+}$



⇒ Electric line of force - connects E vectors that go in the same general direction

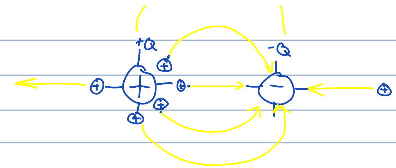
⇒ E is directed away from ⊕ charges

$\boxed{-}$

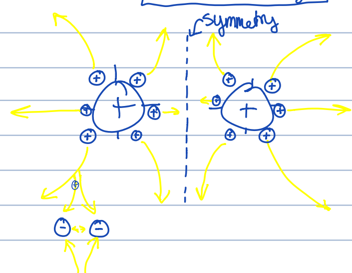


⇒ E vectors always go towards the ⊖ charge

$\boxed{+/-\text{ (dipole)}}$

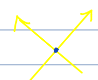


$+/+ \rightarrow$ 2 like charges

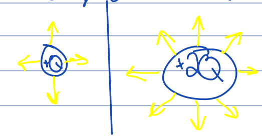


E RULES

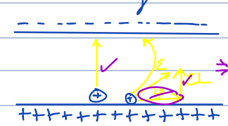
Electric lines of force can't cross, b/c if they did, then the positive charge would have to go in 2 directions at once.



Way to show more charge - 2x amount of lines

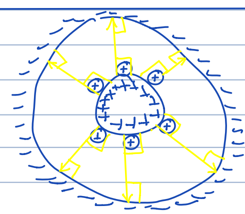


E for a charged surface

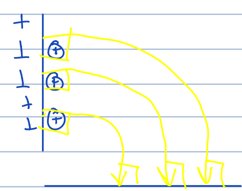


→ E is ⊥ to a charged surface which is at rest because $E_{\parallel}$ would cause charges to move

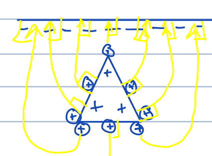
Case 1



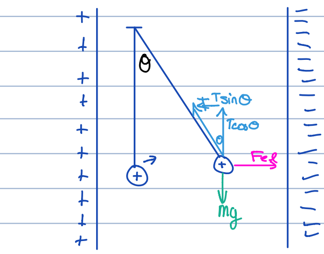
Case 2



Case 3



* E Field Problem 1 - what is θ?



Step1: FBD

Step2/3: N2 and solve

$F_{el} = \frac{k_e Q}{r^2} \rightarrow \mathcal{E} = \frac{F_{el}}{q} \rightarrow F_{el} = q\mathcal{E}$

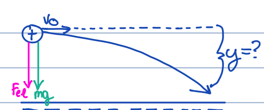
$\rightarrow T\sin\theta = F_{el} = q\mathcal{E} \Rightarrow \tan\theta = \frac{q\mathcal{E}}{mg} \rightarrow \boxed{\theta = \tan^{-1}\left(\frac{q\mathcal{E}}{mg}\right)}$

$\div (T\cos\theta = mg)$

$E = 1000\ \frac{N}{C}$ ← moderate

$E = 1{,}000{,}000\ \frac{N}{C}$ ← air breaks down as an insulator which causes sparks to fly

*E field problem 2 - find y

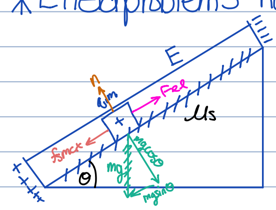


$y = \cancel{v_{0y}t}^{=0} + \frac{1}{2}a_y t^2 \Rightarrow y = \frac{1}{2}a_y t^2 \rightarrow \boxed{y = \frac{1}{2}\left(\frac{q\mathcal{E}}{m} + g\right)\left(\frac{x}{v_0}\right)^2}$

$x = v_{0x}t \rightarrow x = v_0 t \rightarrow \boxed{t = \frac{x}{v_0}}$

$F = ma \rightarrow ma_y = F_{el} + mg = q\mathcal{E} + mg \rightarrow \boxed{a_y = \frac{q\mathcal{E}}{m} + g}$

*E field problem 3 - how much E do you need to just start mass moving?



Step1: FBD

Step2/3: N2 and solve

$F_{el} = mg\sin\theta + f_{smax}$

$qE = mg\sin\theta + \mu_s mg\cos\theta$

$\boxed{E = \frac{mg\sin\theta + \mu_s mg\cos\theta}{q}}$